Minitest 1 - MTH 1410

Dr. Adam Graham-Squire, Fall 2017 9.5 min → 30

Name: Key

I pledge that I have neither given nor received any unauthorized assistance on this exam.

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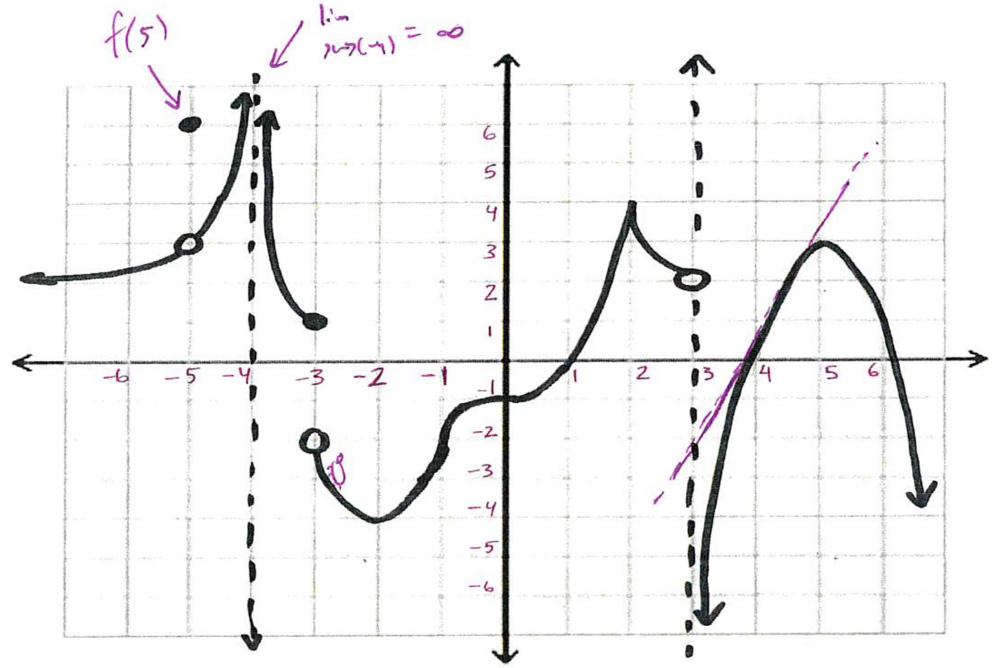
(*signature*)

DIRECTIONS

1. Don't panic.
2. Show all of your work and use correct notation. A correct answer with insufficient work or incorrect notation will lose points.
3. Clearly indicate your answer by putting a box around it.
4. Cell phones and computers are not allowed on this test. Calculators are allowed on the first 5 questions of the test, however you should still show all of your work. No calculators are allowed on the last question of the test.
5. Give all answers in exact form, not decimal form (that is, put π instead of 3.1415, $\sqrt{2}$ instead of 1.414, etc) unless otherwise stated.
6. Make sure you sign the pledge above.
7. Number of questions = 5. Total Points = 25.

1. (6 points) For the following graph of $f(x)$, answer the questions below. If something does not exist, your answer should be ∞, $-\infty$, or DNE, whichever is most appropriate.



(a) $\lim\limits_{x \to (-3)^+} f(x) =$ -2

(b) $f(-5) =$ 6

(c) $\lim\limits_{x \to (-5)} f(x) =$ 3

(d) $f'(4) =$ 3 (It is fine to approximate) (1 to 5 is okay)

(e) Find a number p such that $\lim\limits_{x \to p^-} f(x) = \infty$ If $p = -4$, $\lim\limits_{x \to (-4)^-} f(x) = \infty$

(f) Find one x-value where f is continuous but $f'(x)$ does not exist. (Note: there may be more than one correct answer)

at $x = -1$ ← vert. tangent

at $x = 2$ ← cusp

2. (5 points) Use the definition of the derivative to calculate $f'(2)$ for

$$f(x) = \sqrt{x+7}$$

Note: it is okay to check your work by doing other methods, but you will only receive points for showing your work and using the definition to calculate the derivative.

$$\lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{2+h+7} - \sqrt{2+7}}{h}$$

$$= \lim_{h \to 0} \frac{(\sqrt{h+9} - \sqrt{9})}{h} \cdot \frac{(\sqrt{h+9} + 3)}{(\sqrt{h+9} + 3)}$$

$$= \lim_{h \to 0} \frac{h + \cancel{9} - \cancel{9}}{h(\sqrt{h+9} + 3)}$$

$$= \lim_{h \to 0} \frac{\cancel{h} \; 1}{\cancel{h}(\sqrt{h+9} + 3)}$$

$$= \frac{1}{\sqrt{9} + 3} = \boxed{\frac{1}{6}}$$

3. (4 points) Calculate each limit. Explain your reasoning or show it in a mathematically correct way. If the limit does not exist, explain (briefly) why. You can use a calculator to confirm your answer, but you should be able to answer the question <u>without</u> needing to use a calculator.

(a) $\lim\limits_{x\to\infty} \dfrac{4x^5 - x^3 + 12x}{\pi x^2 - 7x^5}$ $\cdot \dfrac{\left(\frac{1}{x^5}\right)}{\left(\frac{1}{x^5}\right)} = \lim\limits_{x\to\infty} \dfrac{4 - \frac{1}{x^2} + \frac{12}{x^4}}{\frac{\pi}{x^3} - 7}$ $\to 0$

$= \boxed{\dfrac{4}{-7}}$

$\frac{0}{0}$ Rats!

(b) $\lim\limits_{x\to 4} \dfrac{x^2 - 13x + 36}{x^2 - 16} = \lim\limits_{x\to 4} \dfrac{\cancel{(x-4)}(x-9)}{\cancel{(x-4)}(x+4)}$

$= \dfrac{4-9}{4+4} = \boxed{\dfrac{-5}{8}}$

4. (4 points) Calculate each limit. Use a table of values, graph, or other reasoning to calculate the following limits. If the limit does not exist, give your answer as ∞, $-\infty$, or DNE and explain (briefly) why. In any case, make sure to explain how you get your answer with either mathematical symbols or words.

(a) $\lim_{x \to (-\infty)} \frac{x^3}{e^x}$ $= \frac{(-\infty)^3}{e^{-\infty}} = (-\infty)^3 \cdot e^{\infty} = \boxed{-\infty}$

(b) $\lim_{x \to 0} \frac{\tan x}{x}$ $\Rightarrow$

$$\frac{\tan(0.01)}{\cancel{\tan}\,(0.01)} = 1.00003$$

$$\frac{\tan(0.001)}{0.001} \approx 1$$

$$\frac{\tan(-0.001)}{-0.001} \approx 1 \Rightarrow \boxed{1}$$

No Calculator Name: Key

5. (6 points) Let $f(x) = \begin{cases} \dfrac{1}{x-2} - \dfrac{2}{x(x-2)} & \text{if } x < 2 \\ x - c^2 & \text{if } x \geq 2 \end{cases}$

(a) For what value(s) of c will $f(x)$ be continuous at $x = 2$? Make sure to show/explain your work and use the definition of continuity as part of your explanation.

Need $\lim_{x\to 2} f(x) = f(2)$ $\qquad f(2) = 2 - c^2$

$\lim_{x\to 2^+} f(x) = 2 - c^2$

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} \left(\frac{x}{x}\right)\frac{1}{x-2} - \frac{2}{x(x-2)}$

$= \lim_{x\to 2^-} \frac{(x \cancel{-2})}{x(x\cancel{-2})}$

$= \lim_{x\to 2^-} \frac{1}{x} = \frac{1}{2}$

so need these equal

$\Rightarrow \frac{1}{2} = 2 - c^2 \Rightarrow c^2 = \frac{3}{2} \Rightarrow \boxed{c = \pm\sqrt{\frac{3}{2}}}$

(b) For what x-value will $f(x)$ be discontinuous (no matter what the value of c is)?

at $x = 0$, $\frac{2}{x(x-2)}$ is undefined,

$\Rightarrow$ $f(0)$ dne and $f(x)$ ~~also~~

$\boxed{\text{is discontinuous at } x = 0}$

Extra Credit(1 point) Use the limit definition of the derivative to calculate $f'(x)$ if $f(x) = 7$.

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{7 - 7}{h}$$

$$= \lim_{h\to 0} \frac{0}{h}$$

$$= \lim_{h\to 0} 0 = \boxed{0}$$